EMI
المدرسة المحمدية للمهندسين
Ecole Mohammadia d'Ingénieurs
جامعة محمد الخامس بالرباط
Université Mohammed V de Rabat

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Optimisation de la forme des poutres à treillis métallique / Topology and Geometry Optimization of Trusses

Encadrant / Professeur : Mme. EL MKHALET Mouna

Réalisé par : AMINE BENHAMMOU

Optimisation de la forme des poutres à treillis métallique / Topology and Geometry Optimization of Trusses

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PROJET D'OPTIMISATION : Topology and Geometry Optimization of Trusses

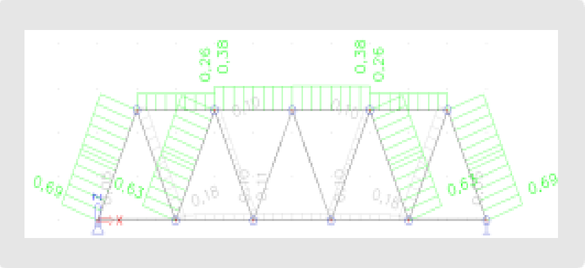
1. Introduction :

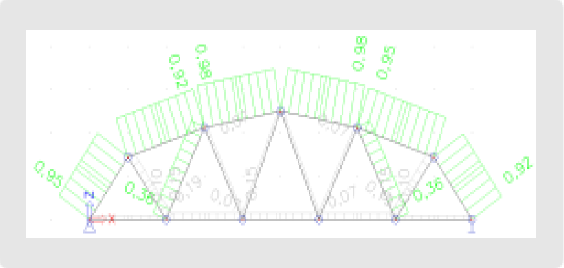
Structural optimization has made important progress in recent years and is now recognized as a practical design tool . Truss optimization can be considered from three distinct perspectives. Topology optimization deals with selection of nodes and their connectivity. On another level, shape optimization seeks to find the optimal coordinates of existing nodes. Finally, cross-sections of truss members can be optimized, which is known as size optimization. In each case, the optimization problem is subjected to some constraints on nodal displacements, member stresses, critical buckling loads, natural frequencies, etc. The objective function is usually set to minimize the structure weight which usually correlates with the overall cost. While methods for shape and size optimization are quite similar in available truss optimizers, a great variety of ideas for topology optimization have been developed in recent decades .

The history of shape and topology optimization of framed structures can be classified as the next classification , The initial period during which Michell (1904) and Maxwell (1894) made their pioneering studies in the field; Although the study by Michell is important in view of theoretical background, the techniques apply only to limited types of discrete structures and constraints (Hemp 1973). Following these initial works, the field of structural topology optimization fell relatively dormant for many decades A second period occured during the 1960's and 1970's in which time interest in structural optimization was re-kindled by the initial developments of high-speed computers. During this period, many important theoretical results for general optimization methods and numerical implementations were first presented, and difficulties in structural topology optimization were given extensive attention. In addition, methods for discrete shape/topology optimization were exercized on very small test problems due to computing limitations . A third period during the 1980's and 1990's has been characterized by extremely dramatic growth in computing technologies. While theoretical work on structural topology optimization has continued, numerical techniques for structural topology optimization have been further refined, developed, and applied to larger-scale, more realistic structures. During this third period, while advances in discrete topology optimization continued, novel continuum structural topology optimization methods were also introduced and developed quite extensively. (you can find more details in the link bellow

https://www.researchgate.net/publication/326377131_Truss_Topology_Optimization_A_Review)

So this research will deal with geometry and topology optimization of truss structures. It presents an approach to optimize simultaneously the geometry and topology of statically undetermined trusses considering the acting forces and the yielding stress of the bars as random variables





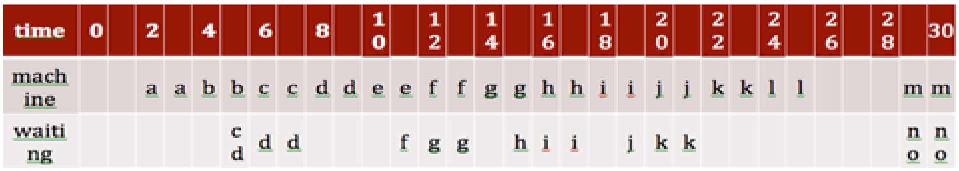
2. difference between deterministic and probabilistic optimization

A **deterministic** optimization does not include elements of randomness. Every time you run the model with the same initial conditions you will get the same results.
Most simple mathematical models of everyday situations are deterministic, for example, the height (h) in metres of an apple dropped from a hot air balloon at 300m could be modelled by $h = - 5t^2 + 300$, where t is the time in seconds since the apple was dropped.

Simple statistical statements, which do not mention or consider variation, could be viewed as deterministic models. The linear regression equation in a bivariate analysis could be applied as a deterministic model if, for example, lean body mass = 0.8737(body weight) - 0.6627 is used to determine the lean body mass of an elite athlete.

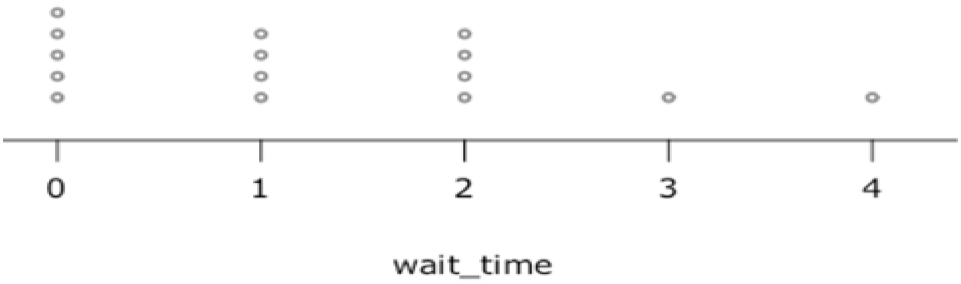
A **probabilistic** optimization includes elements of randomness. Every time you run the model, you are likely to get different results, even with the same initial conditions. A probabilistic model is one which incorporates some aspect of random variation.

Exemple of probabilistic optimization



time	0		2		4		6		8		10		12		14		16		18		20		22		24		26		28		30
machine			a	a	b	b	c	c	d	d	e	e	f	f	g	g	h	h	i	i	j	j	k	k	l	l				m	m
waiting						c d	d	d				f	g	g		h	i	i		j	k	k								n o	n o

e distribution of waiting times from the simulation is:



3. Deterministic optimization :

3.1. Formulation of the optimization problem :

The optimization problem is posed as the minimization of the volume of the structure subject to stress constraints by taking the nodal coordinates and cross-section areas as design variables: Find: x and A

that gives: min (x, A) A .L(x) T V = , (1)

subject to g (j , ,...,m) j = +σ j −σ t ≤ 0 = 1 2 , (2)

g (j , ,...,m) j +m = −σ j +σ c ≤ 0 = 1 2 . (3)

where V is the volume of the structure, x is the vector of nodal coordinates, A is the vector of member areas, L is the vector of member lengths, gj are stress constraints, σj is the stress on member j, σt is the yielding stress in tension, σc is the yielding stress in compression and m is the number of members subjected to stress constraints. In this article, buckling constraints are not introduced.

For convenience, the design variables A and x can be grouped into a single design vector X, and the constraints from Eq. (2) and Eq. (3) can be grouped into a single vector of constraints g. In this way, the previous problem is rewritten as follows:

Find: X that gives min (X) A .L T V = , (4)

subject to g ≤ 0 , (5)

where g is a vector with 2m components since there are two constraints defined for each bar of the structure.

3.2. Sensitivity analysis :

Most nonlinear optimization methods need to evaluate the gradients of both the objective function and constraints with respect to the design variables. The partial derivative of the volume V with respect to each member area is given by the length of each bar. The partial derivative of V with respect to the nodal coordinates is also easy to obtain, and expressions similar to the ones presented by Martínez et al. (2007) and Torii and Biondini (2009) can be used for this purpose. Moreover, all these derivatives can be efficiently evaluated by finite differences, if necessary, since the evaluation of V is not computationally demanding.

The major difficulty arises when evaluating the gradients of the constraints from Eq. (2) and Eq. (3), since these constraints involve stresses that depend implicitly on nodal displacements. From Eq. (2) and Eq. (3) it can be seen that the partial derivatives from these expressions will be the same with opposite sign and, consequently, only one of them must be evaluated numerically.

Working with an arbitrary chosen constraint gj , as defined in Eq. (2), its partial derivative with respect to a design variable Xi can be written as :

$$\frac{\partial g_j}{\partial X_i} = \frac{d\sigma_j}{dX_i} + \frac{\partial \sigma_j}{\partial \mathbf{u}}^T \cdot \frac{\partial \mathbf{u}}{\partial X_i},$$

where the symbol d stands for an ordinary derivative and the symbol ∂ stands for a partial derivative. It is worth noting that the first term in Eq. (6), the derivative dσj/dXi , is a derivative related to the direct dependence of σj on Xi . The second term, instead, is an implicit derivative related to the dependence of σj on Xi through the nodal displacements u.

The partial derivative ∂σj/∂u can be evaluated analytically, or found by finite differences if necessary, since the evaluation of σj when only displacements are changed is straightforward. However, the partial derivative ∂u/∂Xi is difficult to evaluate, and its computation should be avoided. This can be accomplished by using the adjoint method, as described by Haftka and Gurdal (1992).

3.3. Adjoint method :

The following system of linear equations relating the applied forces F and the displacements u is considered (Bathe 1996):

$$\mathbf{K}.\mathbf{u} = \mathbf{F}, \qquad (7)$$

where K is the stiffness matrix of the structure. The partial derivative of Eq. (7) with respect to a design variable is :

$$\frac{\partial \mathbf{K}}{\partial X_i}.\mathbf{u} + \mathbf{K}.\frac{\partial \mathbf{u}}{\partial X_i} - \frac{\partial \mathbf{F}}{\partial X_i} = \mathbf{0},$$

which has been rearranged in order to give an expression equal to a zero vector. Since Eq. (8) is equal to a zero vector, Eq. (6) can be rewritten as :

$$\frac{\partial g_j}{\partial X_i} = \frac{d\sigma_j}{dX_i} + \frac{\partial \sigma_j}{\partial \mathbf{u}}^T.\frac{\partial \mathbf{u}}{\partial X_i} - \boldsymbol{\tau}_j^T.\left(\frac{\partial \mathbf{K}}{\partial X_i}.\mathbf{u} + \mathbf{K}.\frac{\partial \mathbf{u}}{\partial X_i} - \frac{\partial \mathbf{F}}{\partial X_i}\right) \qquad (9)$$

or

$$\frac{\partial g_j}{\partial X_i} = \frac{d\sigma_j}{dX_i} + \left(\frac{\partial \sigma_j}{\partial \mathbf{u}}^T - \boldsymbol{\tau}_j^T.\mathbf{K}\right).\frac{\partial \mathbf{u}}{\partial X_i} - \boldsymbol{\tau}_j^T.\left(\frac{\partial \mathbf{K}}{\partial X_i}.\mathbf{u} - \frac{\partial \mathbf{F}}{\partial X_i}\right), \qquad (10)$$

where τj is an arbitrary vector. Eq. (10) shows that the evaluation of ∂u/∂Xi can be avoided if

$$\frac{\partial \boldsymbol{\sigma}_j}{\partial \mathbf{u}}^T - \boldsymbol{\tau}_j^T.\mathbf{K} = \mathbf{0}. \qquad (11)$$

Thus, the vector $\boldsymbol{\tau}_j$ is chosen as the solution of the system of linear equations

$$\mathbf{K}.\boldsymbol{\tau}_j = \frac{\partial \sigma_j}{\partial \mathbf{u}}, \qquad (12)$$

where the symmetry of the stiffness matrix has been used (Bathe 1996). Equation (10) then becomes

$$\frac{\partial g_j}{\partial X_i} = \frac{d\sigma_j}{dX_i} - \boldsymbol{\tau}_j^T.\left(\frac{\partial \mathbf{K}}{\partial X_i}.\mathbf{u} - \frac{\partial \mathbf{F}}{\partial X_i}\right), \qquad (13)$$

that can be easily evaluated once the vector τj is found from Eq. (12). In fact, the derivatives ∂K/∂Xi , and ∂F/∂Xi can be evaluated analytically (Torii and Biondini 2009), or numerically by finite differences. It is worth noting that the evaluation of the partial derivative ∂u/∂Xi could be also avoided by using Eq. (8) rewritten as follows:

$$\frac{\partial \mathbf{u}}{\partial X_i} = \mathbf{K}^{-1}.\left(\frac{\partial \mathbf{F}}{\partial X_i} - \frac{\partial \mathbf{K}}{\partial X_i}.\mathbf{u}\right). \qquad (14)$$

However, in this approach the number of systems of linear equations to be solved is equal to the number of design variables. In the adjoint method, instead, the number of systems of linear equations is equal to the number of bars in the ground structure. Since the number of design variables is the number of members (cross section areas) plus the number of nodal coordinates taken as design variables, the adjoint method will always lead to a lower number of systems of linear equations to be solved.

Finally, it is outlined that if the constraints and the objective function of the optimization problem are both incorporated into a single objective function, by means of the augmented Lagrangean method or some other penalty methods (Rao, 1996), the number of systems of linear equations to be solved during sensitivity analysis can be drastically reduced, as demonstrated by Pereira et al. (2004).

3.4. Alternative loading conditions :

When a set of s alternative loading conditions {F1 ,...,Fk ,...,Fs} is considered, the structural response will be defined by a set of nodal displacements {u1 ,...,uk ,...,us}, and the optimization problem from Eq. (4) and Eq. (5) becomes

Find: X

that gives

$$\min V(\mathbf{X}) = \mathbf{A}^T \mathbf{L}, \qquad (15)$$

subject to

$$\mathbf{g}_k \leq \mathbf{0} \quad (k = 1,\dots,s), \qquad (16)$$

-where the vectors gk from Eq. (16) are defined for each loading condition in Eq. (2) and Eq. (3).
-Working again with constraints related to the yielding stress in compression, since constraints related to tension will just have opposite sign, the derivative of constraint gjk, related to the stress in the bar j when the loading condition k is applied, is :

$$\frac{\partial g_{jk}}{\partial X_i} = \frac{d\sigma_{jk}}{dX_i} + \frac{\partial \sigma_{jk}}{\partial \mathbf{u}_k}^T . \frac{\partial \mathbf{u}_k}{\partial X_i}. \qquad (17)$$

The adjoint vector is now given by the solution of the system of linear equations

$$\mathbf{K}.\boldsymbol{\tau}_{jk} = \frac{\partial \sigma_{jk}}{\partial \mathbf{u}_k} \qquad (18)$$

and Eq. (17) can be rewritten as

$$\frac{\partial g_{jk}}{\partial X_i} = \frac{d\sigma_{jk}}{dX_i} - \boldsymbol{\tau}_{jk}^T \left(\frac{\partial \mathbf{K}}{\partial X_i} .\mathbf{u}_k - \frac{\partial \mathbf{F}_k}{\partial X_i} \right). \qquad (19)$$

3.5. Bounds on the design variables :

There are two different approaches for defining bounds on the nodal coordinates. In the first approach, bounds can be defined locally for each design variable, as shown in Fig. 1a. In this case, there are different bounds for each design variable, and this can be accomplished by defining a rectangular feasible region around each node. The second approach is that of defining bounds for all the design variables at once, as shown in Fig. 1b. In this case, the bounds are the same for all design variables. These two approaches may lead to different results, since the feasible domain defined in the first approach is smaller. However, the first approach may prevent problems related to node superposition, if the bounds are defined properly. Therefore, this approach may be recommended when there are many nodes for which the coordinates are taken as design variables in the optimization procedure. For members areas it is necessary to allow only positive values, defining in this way a lower bound for these design variables. Upper bounds on members areas are not strictly necessary, since the algorithm will seek a structure with minimum volume. However, defining upper bounds on all design variables may be recommended, since the optimization problem becomes better posed.

4. Probabilistic optimization :

4.1. Formulation of the problem and reliability assessment :

Consider the applied forces F and the yielding stresses σ t or c to be random variables with known density distribution. For convenience of notation, they are grouped into the random variable vector **Z** . The optimization problem still searches for the minimum volume structure, but now subject to a minimum reliability level of the structure. A component level reliability constraint is considered, instead of dealing with the probabilistic failure constraint at the system level. In other words, a minimum reliability level is enforced for each bar. For a single loading condition this leads to the following problem:

Find: X

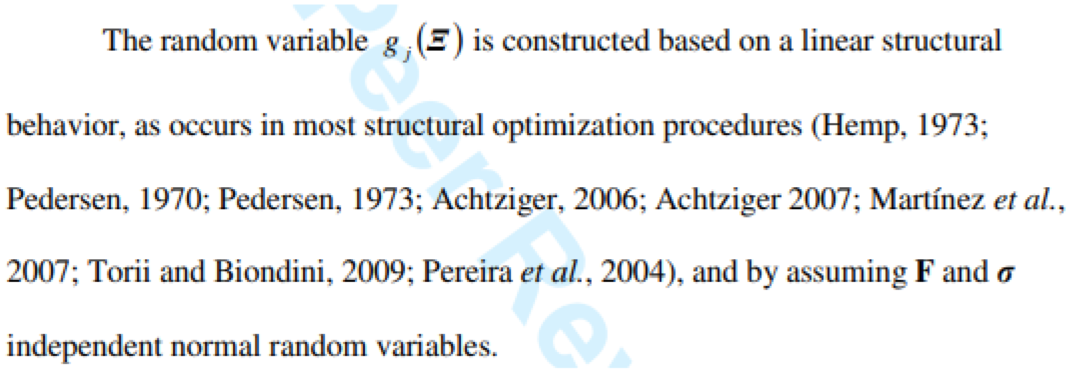
that gives :

$$\min V(\mathbf{X}) = \mathbf{A}^T.\mathbf{L}\,, \tag{20}$$

subject to

$$G_j = P_0 - P\left(g_j(\Xi) \le 0\right) \le 0 \quad (j = 1,\ldots,m)\,, \tag{21}$$

where $P(\cdot)$ is the probability of the constraint to be fulfilled and P_0 is a minimum probability level of the constraint or its reliability. Note that now the constraints of the optimization problem are affected by the random variable vector Ξ , becoming themselves random variables. The constraint from Eq. (21) states that the probability of $g_j(\Xi)$ being respected must be higher than a minimum probability P_0.



The random variable $g_j(\Xi)$ is constructed based on a linear structural behavior, as occurs in most structural optimization procedures (Hemp, 1973; Pedersen, 1970; Pedersen, 1973; Achtziger, 2006; Achtziger 2007; Martínez *et al.*, 2007; Torii and Biondini, 2009; Pereira *et al.*, 2004), and by assuming **F** and $\boldsymbol{\sigma}$ independent normal random variables.

If for a given applied force vector F0, the resulting stress in a given member is σ0, then for an arbitrary applied force vector F, obtained by the multiplication of F0 by a scalar, the stress in that member is (by the principle of superposition from structural mechanics)

$$\sigma = \frac{\sigma_0}{\|\mathbf{F}_0\|}.\|\mathbf{F}\| = \frac{\sigma_0}{F_0}.F, \qquad (22)$$

Where|| · ||denotes the norm of a vector, and F and F0 are the norms of F and F0, respectively. Equation (22) can be substituted into Eq. (2), giving

$$g(\Xi) = \frac{\sigma_0}{F_0}.F - \sigma_t \le 0, \qquad (23)$$

where the index j has been dropped for convenience.

Since both the applied forces and yielding stresses are considered as independent normal random variables, a linear combination of them is also a normal random variable. Consequently, by denoting µ1 and s1 the mean value and standard deviation of the applied force, respectively, and µ2 and s2 the mean value and standard deviation of the yielding stress, respectively, g(**Z**) as given by Eq. (23) is a normal random variable with mean

$$\mu_{g(\Xi)} = \frac{\sigma_0}{F_0}.\mu_1 - \mu_2 \qquad (24)$$

and standard deviation

$$s_{g(\Xi)}^2 = \left(\frac{\sigma_0}{F_0}\right)^2.s_1^2 + s_2^2. \qquad (25)$$

The normalized value of g(**Z**) is then :

$$\beta = \frac{g(\Xi) - \mu_{g(\Xi)}}{s_{g(\Xi)}}, \qquad (26)$$

where β is the reliability or also called reliability index of the constraint (Lemaire et al., 2005). Thus, the reliability index is related to the probability of the constraint being feasible by:

$$\beta = \Phi^{-1}\left[P\left(g_j(\Xi) \le 0\right)\right] \text{ or } P\left(g_j(\Xi) \le 0\right) = \Phi[\beta], \tag{27}$$

where Φ = Φ[•] is the standard normal cumulative probability function. The required minimum reliability level of the structure P0 is related to the so-called target reliability index by:

$$P_0 = \Phi[\beta_t] \text{ or } \beta_t = \Phi^{-1}[P_0]. \tag{28}$$

Substituting Eq. (27) and (28), the constraint given by Eq. (21) becomes

$$G_j = \beta_t - \beta_j \le 0 \quad (j = 1, \ldots, m). \tag{29}$$

The index j emphasizes that a reliability index is computed for each bar.

4.2. Sensitivity analysis :

The derivative of Eq. (29) with respect to a design variable is given by :

$$\frac{\partial G_j}{\partial X_i} = -\frac{d\beta_j}{dX_i} - \frac{\partial \beta_j}{\partial \mathbf{u}}^T \cdot \frac{\partial \mathbf{u}}{\partial X_i}, \tag{30}$$

where the derivative of the constraint with respect to a given design variable is composed of an explicit plus an implicit derivative, as occurs for the deterministic case. The ordinary derivative dβj/dXi can be evaluated by finite differences, since only perturbations to the design variables are necessary. The second part of Eq. (30) can be solved efficiently by applying the adjoint method again, in order to avoid the evaluation of the derivative ∂u/∂Xi . Following the same procedure described for the deterministic approach:

$$\frac{\partial G_j}{\partial X_i} = -\frac{d\beta_j}{dX_i} + \boldsymbol{\tau}_j^T \cdot \left(\frac{\partial \mathbf{K}}{\partial X_i} \cdot \mathbf{u} - \frac{\partial \mathbf{F}}{\partial X_i} \right), \tag{31}$$

where the adjoint vector $\boldsymbol{\tau}_j$ is found by solving

$$\boldsymbol{K} \cdot \boldsymbol{\tau}_j = -\frac{\partial \beta_j}{\partial \boldsymbol{u}}. \tag{32}$$

4.3. Loading conditions given by several forces with different standard deviations :

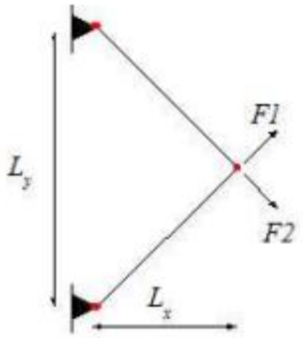


Figure 2: Example of a structure subject to one loading condition given by two forces with different standard deviations.

In order to describe how forces with different standard deviations can be taken into account into the same loading condition, consider the example from Fig. 2. In this case, the structure is subjected to a single loading condition. That is, the forces F1 and F2 are applied at the same time, but have different mean values and standard deviations. In order to access the reliability of the structure, it is necessary to study the effect of each force, F1 and F2 , separately. Thus, considering each force separately (again by the superposition principle)

$$\sigma^1 = \frac{\sigma_0^1}{\left\|\mathbf{F}_0^1\right\|}.\left\|\mathbf{F}^1\right\| = \frac{\sigma_0^1}{F_0^1}.F^1 \qquad (33)$$

and

$$\sigma^2 = \frac{\sigma_0^2}{\left\|\mathbf{F}_0^2\right\|}.\left\|\mathbf{F}^2\right\| = \frac{\sigma_0^2}{F_0^2}.F^2 , \qquad (34)$$

where the same notations as of Eq. (22) hold, but here the superscripts 1 and 2 represent the quantities for forces F1 and F2 . That is, one structural analysis is made for each force F01 and F02 , which gives the stresses σ 01 and σ 02 , respectively (usually, these forces are taken as unit forces, for convenience). The stress caused by the application of F1 + F2 is then

$$\sigma = \sigma^1 + \sigma^2 = \frac{\sigma_0^1}{F_0^1}.F^1 + \frac{\sigma_0^2}{F_0^2}.F^2 , \qquad (35)$$

which can be rewritten, for convenience, as

$$\sigma = k_1.F^1 + k_2.F^2 . \qquad (36)$$

The important aspect here is that the effect inside the bar can be written as a linear combination of the separate effects of each force, as this is a linear structural analysis. This is precisely the superposition principle from structural mechanics. Since the applied forces are Gaussian random variables, the mean value of the stress in the bar and its standard deviation due to the combination of forces F1 and F2 (Eq. (36)) are, respectively,

$$\sigma^1 = \frac{\sigma_0^1}{\left\|\mathbf{F}_0^1\right\|}.\left\|\mathbf{F}^1\right\| = \frac{\sigma_0^1}{F_0^1}.F^1 \tag{33}$$

and

$$\sigma^2 = \frac{\sigma_0^2}{\left\|\mathbf{F}_0^2\right\|}.\left\|\mathbf{F}^2\right\| = \frac{\sigma_0^2}{F_0^2}.F^2\,, \tag{34}$$

where the same notations as of Eq. (22) hold, but here the superscripts 1 and 2 represent the quantities for forces F1 and F2 . That is, one structural analysis is made for each force F_0^1 and F_0^2 , which gives the stresses σ_0^1 andσ_0^2, respectively (usually, these forces are taken as unit forces, for convenience). The stress caused by the application of F1 + F2 is then

$$\sigma = \sigma^1 + \sigma^2 = \frac{\sigma_0^1}{F_0^1}.F^1 + \frac{\sigma_0^2}{F_0^2}.F^2\,, \tag{35}$$

which can be rewritten, for convenience, as

$$\sigma = k_1.F^1 + k_2.F^2\,. \tag{36}$$

The important aspect here is that the effect inside the bar can be written as a linear combination of the separate effects of each force, as this is a linear structural analysis. This is precisely the superposition principle from structural mechanics. Since the applied forces are Gaussian random variables, the mean value of the stress in the bar and its standard deviation due to the combination of forces F1 and F2 (Eq. (36)) are, respectively,

$$\mu_\sigma = k_1.\mu_{F^1} + k_2.\mu_{F^2} \tag{37}$$

and

$$(s_\sigma)^2 = \left(k_1.s_{F^1}\right)^2 + \left(k_2.s_{F^2}\right)^2, \tag{38}$$

where μ_{F1} and S_{F1} are the mean value and standard deviation of the force F 1 , respectively (the same notation is also used for the force F 2). Moreover, $\mu\sigma$ and σ s are the mean value and the standard deviation of the stress inside the bar, respectively. Note that the stress is a Gaussian random variable, as it is the linear combination of two Gaussian random variables, namely the two forces. Thus far it has been shown how to obtain the mean value and the standard deviation of the stress inside the bar, given two forces with magnitude F 1 and F 2 acting simultaneously. Consider now the stress constraint from Eq. (2). Since it is a linear combination of Gaussian random variables (the stress and the maximum allowable stress),

$$\mu_{g(\Xi)} = \mu_{\sigma} - \mu_{\sigma_t} \tag{39}$$

and

$$s_{g(\Xi)}^{\ 2} = \left(s_{\sigma}\right)^2 + \left(s_{\sigma_t}\right)^2 \tag{40}$$

where µ g (**Z**) and s g (**Z**) are the mean and the standard deviation of the constraint, and **µ** $_{\sigma t}$ and S$_{\sigma t}$ are the mean and the standard deviation of the maximum allowable stress in tension. It is possible to evaluate Eq. (39) and Eq. (40) with Eq. (37) and Eq. (38), and consequently, to obtain the reliability index as described previously. Also, note that Eq. (39) and Eq. (40) are analogous to Eq. (24) and Eq. (25), respectively. The previous results can also be extended to more than two forces acting at the same time. However, this leads to a significant increase in the number of structural analysis that must be made. Also, the sensitivity analysis is also changed, since now there will be one adjoint vector associated with each force.

5. Numerical examples :

In this section, several numerical examples are solved in order to discuss the main aspects of the approach presented. Note that for all the following examples each figure has its own scale, and the reader can compare the different solutions by the volume of material V presented for each example. A lower bound for the areas is defined as 1E-10m^2 (i.e. 0.1mm^2), in order to avoid singularity of the stiffness matrix. Finally, the optimization algorithm used here is the Sequential Quadratic Programming (SQP) (Nocedal and Wright, 1999; Rao, 1996)

5.1 Example 1 – Effect of the reliability index

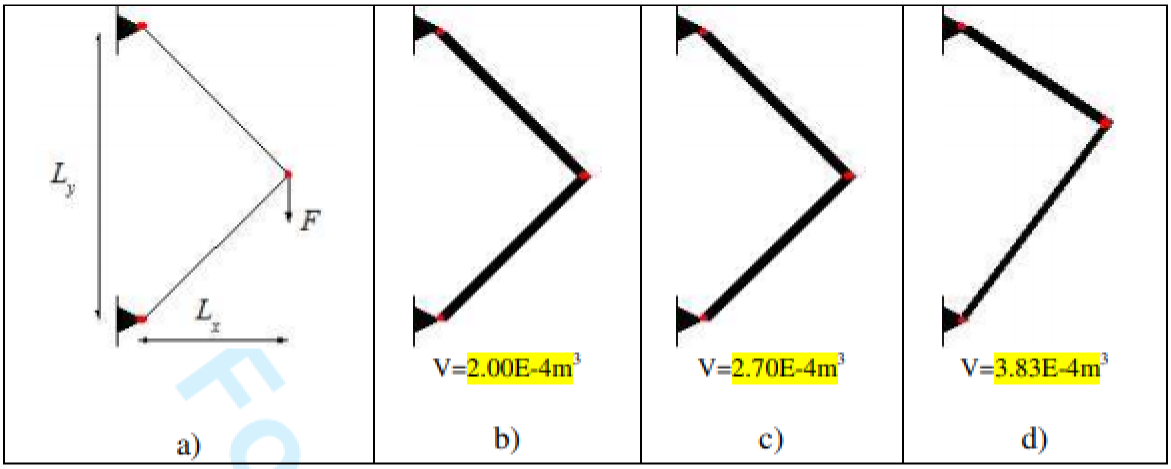


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Figure 3: a) Ground structure, b) optimum solution for the deterministic problem, c) optimum solution with yielding stresses in tension and compression with the same standard deviation and d) optimum solution with yielding stress in tension with a higher standard deviation.

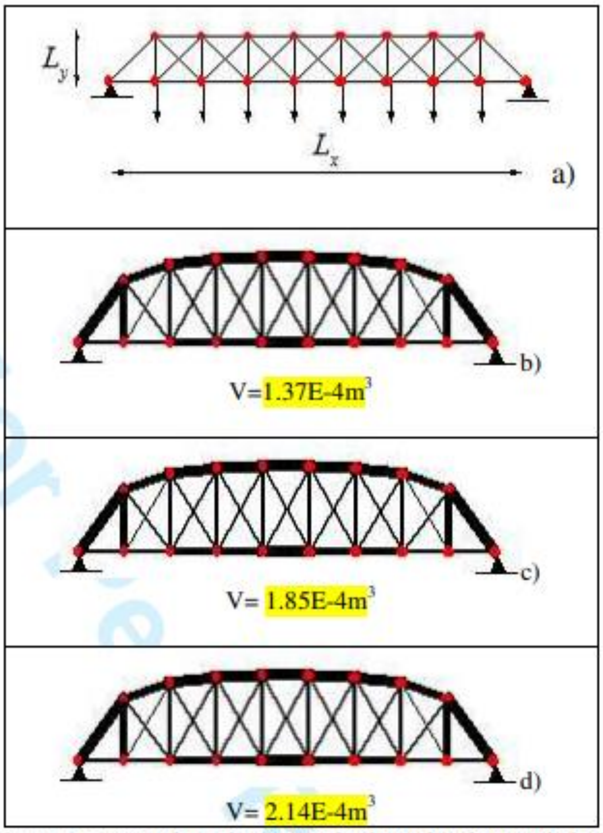


Figure 4: a) Ground structure and optimum solutions b) deterministic problem, c) β equal to 3.1 and d) β equal to 4.75.

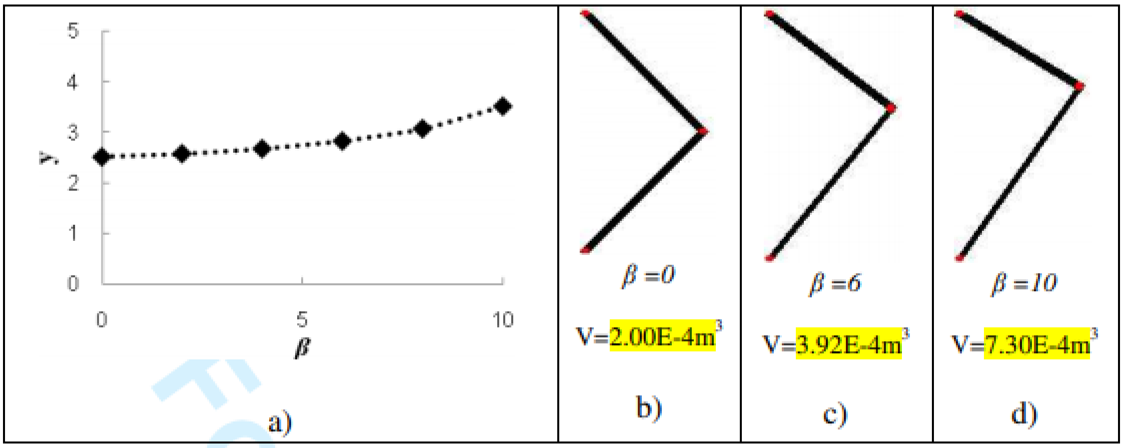
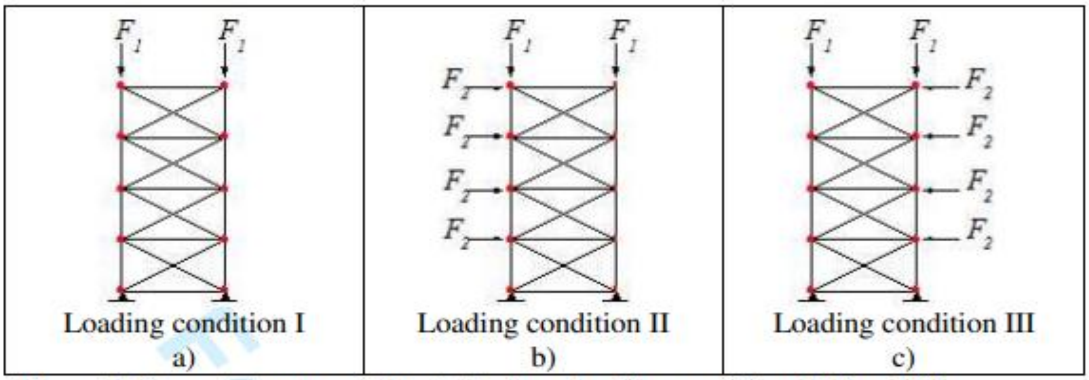


Figure 5: a) The change of the y coordinate of the central node of the optimal design for different reliability indexes and b) c) d) some chosen optimum solutions.

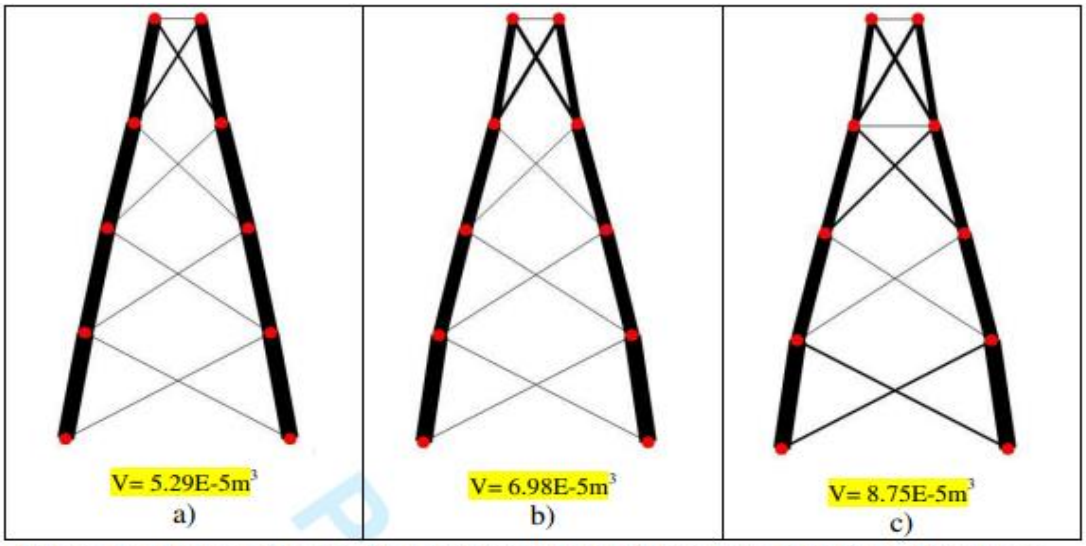
In order to show that increasing the reliability index may lead to geometry and topology changes, return to the example from **Fig. 3**. The loads and dimensions of the ground structure remain unchanged as well as most of the material properties. The only modification is that the standard deviation of the yielding stress in compression is now equal to $\sigma_{\sigma c}$ = 10MPa and of the yielding stress in tension equal to $\sigma_{\sigma t}$ = 20MPa. The problem is then solved for reliability indexes equal to 0 (deterministic problem), 2, 4, 6, 8 and 10. The results are presented in **Fig. 5**. It can be seen that increasing the reliability index leads, in this case, to changes in both topology and geometry. This example contradicts the assumption of only scaling the cross-section areas when different reliability indexes are used. The reason why the example from **Fig. 4** gave scaled solutions for different reliability indexes lie in the fact that all the properties of that ground structure are symmetric. The mean values and standard deviation of the yielding stresses are equal in magnitude, together with the applied forces. The case from **Fig. 5**, instead, has different standard deviations for the yielding stresses, and this leads to changes in topology and geometry when the reliability index is changed.

5.2 Example 5 – Loading conditions composed of several forces :



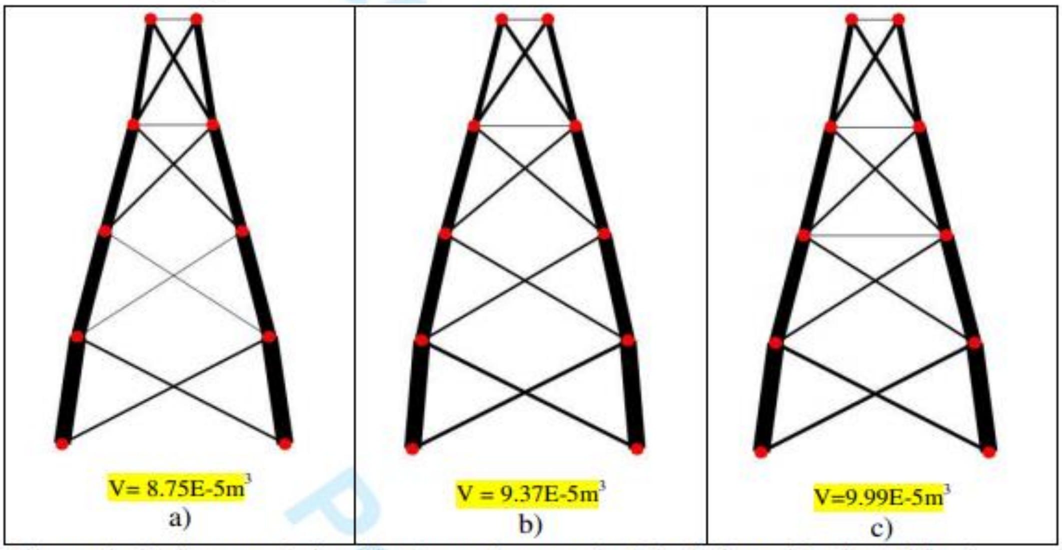
Loading condition I
a)
Loading condition II
b)
Loading condition III
c)

Figure 7: Ground structure subjected to three loading conditions I, II and III.



V= 5.29E-5m^3
a)
V= 6.98E-5m^3
b)
V= 8.75E-5m^3
c)

Figure 8: Optimum solutions for increasing lateral loads from a) to c) for the ground structure from Fig. 8.



V= 8.75E-5m^3
a)
V = 9.37E-5m^3
b)
V=9.99E-5m^3
c)

Figure 9: Optimum solutions for increasing standard deviation of the lateral loads from a) to c) for the ground structure from Fig. 8.

Fig 7 presents a ground structure that is subjected to three loading conditions. However, vertical forces and horizontal forces have different mean values and standard deviations. This example illustrates the application of the method when a loading condition is composed by forces with different mean values and standard deviations.

The ground structure from **Fig. 7** has a total height of 4m and a total width of 2m. The material properties are the same as from example 1. The target reliability index is 3.1, and all nodes (except the nodes of the supports) are allowed to be moved left and right by the optimization algorithm, to positions as far as 0.8m from its original position. The vertical force mean value is F_1 = 1,000N and its standard deviation is σ_{F1} = 100N. For the horizontal forces F_2, three cases are studied: mean equal F_2= 50N and standard deviation σF2 = 5N; mean F2 = 150N and standard deviation σ_{F2} = 15N; and mean F_2 = 250N and standard deviation σ_{F2} = 25N. Finally, symmetry of the geometry is enforced.

The results for this example are presented in **Fig. 8**. As a consequence of the increase to the lateral load, for the same vertical load, there was an increase of the volume of the structure and a change of its geometry and topology. The structures designed for higher lateral loads are clearly more fitted to resisting the increased bending moment that develops in these cases

For the same ground structure from **Fig. 7**, another interesting example can be conceived. Taking the vertical force F_1 as defined previously and the mean of the horizontal force F_2 = 250N, the same problem is now solved for three different standard deviations: σ_{F2} = 25N, σ_{F2} = 50N and σ_{F2} = 75N. The results are presented in **Fig. 9**, where it can be noted that increasing the standard deviation of the lateral load leads to an increase in the volume of the structure. Also, the geometry and the topology change in order to resist the increased bending moments that appear in this case

5.3 Example 3 – Comparison of the sizing and simultaneous sizing-geometry optimization :

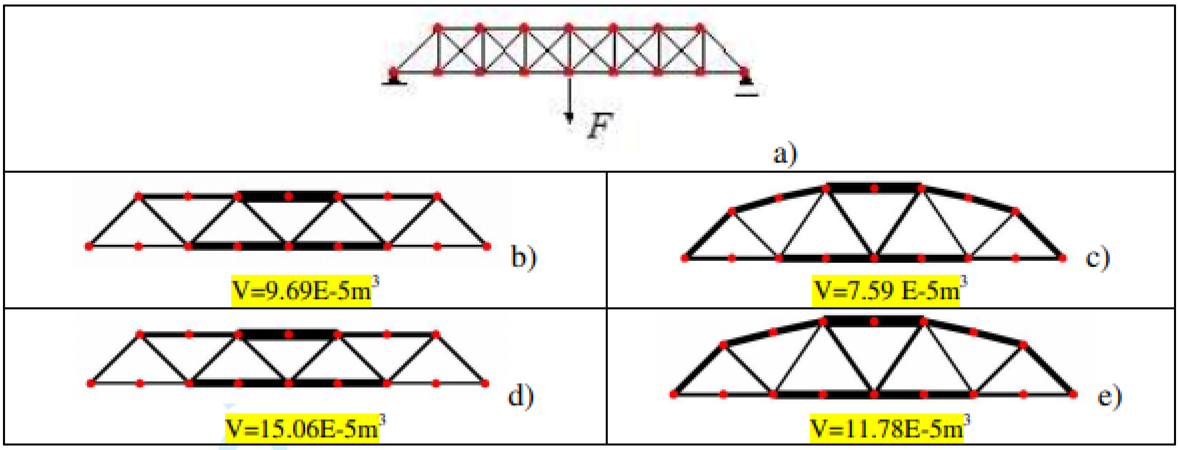


Figure 10: a) Ground structure, b) optimum topology for the deterministic problem, c) optimum geometry for the deterministic problem, d) optimum topology for the probabilistic problem, e) optimum geometry for the probabilistic problem.

The example from **Fig. 10** compares results from the simultaneous optimization of geometry and sizing with results given by sizing optimization only. The ground structure has a total height of 1m and a total length of 8m. The material properties are as defined for the example 1. The target reliability index of the structure is β =4.75, and all the nodes from the upper chord are allowed to be moved up and down, by the optimization algorithm, to positions as far as 0.5m from its original position. The applied force is F = 1,000N with a standard deviation σ_F = 100N. The results from Fig. 10 demonstrate that changes in the geometry of the structure can lead to significant improvements of the solution, even if the topology (the bars that comprise the structure) remains basically the same. This justifies the effort in including the nodal coordinates as design variables of the optimization problem

Further Comments :

There' s a plenty of other problems that is not discussed in the examples befor as (Local minima, Effect of limiting the maximum area of the bars etc ..) Noting that the optimization problems presented in this project have several reliability constraints, and consequently several reliability analysis problems must be solved in order to evaluate a given design point of the optimization process. Here, all these reliability analysis problems are solved analytically, and consequently the main computational effort lies in finding the stresses inside the structures for each applied load .

Conclusions :

This article presented a formulation for the simultaneous optimization of topology and geometry of truss structures. General aspects such as constraints and several loading conditions were also discussed, together with a detailed discussion on sensitivity analysis.

The deterministic optimization scheme was then extended to the case when the yielding stresses and the applied forces are Gaussian random variables. The important aspect here is that considering these variables as Gaussian allows one to access the reliability of the structure directly, without using iterative methods. Also, the sensitivity analysis is also simplified by the use of the adjoint method. Thus, the probabilistic problem as defined here has a similar computational cost per optimization step as the deterministic problem, and that is an important aspect.

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